

Determination of the length and position of a non-visible object (Item No.: P2542001)

Curricular Relevance



Difficulty



Difficult

Preparation Time



1 Hour

Execution Time



2 Hours

Recommended Group Size



2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Bremsstrahlung, characteristic X-radiation, law of absorption, mass absorption coefficient, stereographic projection

Overview

Short description

Principle

This experiment provides training in determining the length and position of an object based on an X-ray image. A metal pin that is embedded in a wooden block is used as the model. This experiment is also an excellent preparatory exercise for demonstrating the principle of computed tomography.



Equipment

Position No.	Material	Order No.	Quantity
1	XR 4.0 expert unit, X-ray unit, 35 kV	09057-99	1
2	XR 4.0 X-ray plug-in unit W tube	09057-81	1
3	XR 4.0 X-ray Implant model	09058-07	1
4	Vernier calliper stainless steel 0-160 mm, 1/20	03010-00	1
5	XR 4.0 X-ray fluorescent screen	09057-26	1
6	Slide mount for optical bench expert, h = 30 mm	08286-01	2
7	Table with stem	09824-01	1
8	XR 4.0 X-ray optical bench	09057-18	1
9	XR 4.0 X-ray Adapter for digital camera 1/4"	09057-15	1
10	XR 4.0 X-ray external optical bench	09057-21	1
11	XR 4.0 X-ray slide for external optical bench	09057-29	1

Tasks

1. X-ray the implant model in two planes that are shifted by 90° with regard to each other. Take a picture of the image on the fluorescent screen.
2. Calculate the true length of the embedded metal pin by taking into account the magnification factor that needs to be determined.
3. Determine the spatial position of the metal pin.

Set-up and procedure

Procedure

Place the implant model directly in front of the fluorescent screen with both of them as far to the right as possible on the optical bench. The distance between the front of the model and the outlet tube of the X-ray plug-in unit is then approximately 30cm . Do not use a diaphragm tube for the irradiation.

- Adjust an acceleration voltage $U_A = 35\text{ kV}$ and an anode current $I_A = 1\text{ mA}$.
- Fasten the camera to the slide mount on the optical bench.
- Select the night mode and deactivate the flash.
- Either darken the room completely or cover the device with the protective cover.
- We recommend taking the picture with the automatic camera release (self-timer) in order to prevent the camera from shaking.
- Then, turn the implant model by 90° around its longitudinal axis and repeat the procedure.



Fig. 2: Set-up

Theory and evaluation

Task 1: X-ray the implant model in two planes that are shifted by 90° with regard to each other. Take a picture of the image on the fluorescent screen.

Figure 3 shows example photos of the implant model.

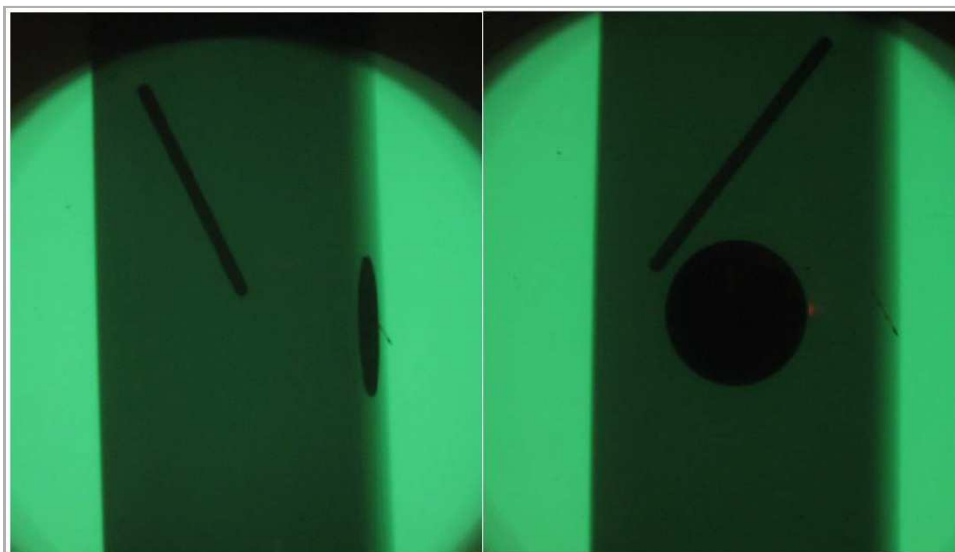


Fig. 3: Pictures of the implant model; projection in the x,z-plane (left) and in the y,z-plane (right)

Task 2: Calculate the true length of the embedded metal pin by taking into account the magnification factor that needs to be determined.

Figure 4 shows a random oblique position of the metal pin of the length l in a three-dimensional space.

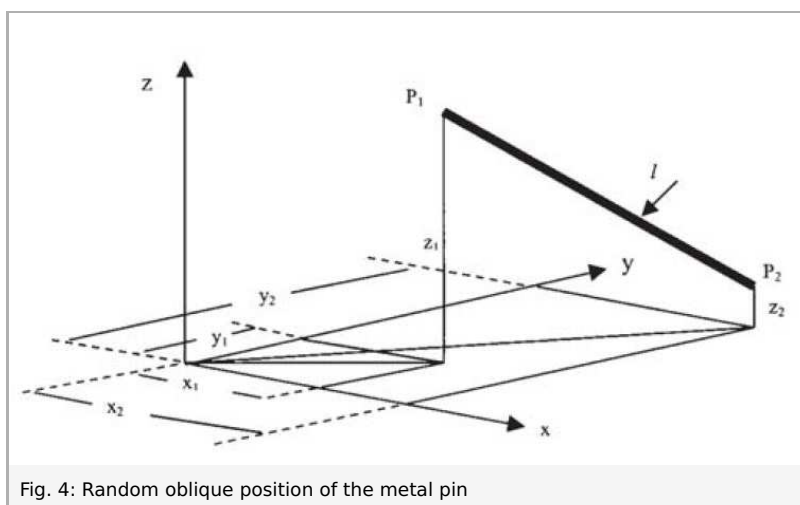


Fig. 4: Random oblique position of the metal pin

The length l of the pin with its ends $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is:

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{l_x^2 + l_y^2 + l_z^2} \quad (1)$$

Since the metal pin is irradiated by a conical bundle of X-rays and since it has a certain distance towards the film plane, it is projected on the film plane in a magnified form. In order to be able to determine the degree of magnification, the implant model is equipped with a metallic reference disk with a diameter of $d = 30 \text{ mm}$. If the diameter of the disk projection on the film is d^* , then the magnification is $V = d^*/d$. As a consequence, the true length of the metal pin is $l_w = l/V$.

Figure 5 shows the projections of the metal pin for two planes of the implant model that are shifted by 90° with regard to each other. For the evaluation in accordance with figure 5, we recommend printing the photo as large as possible and determining the corresponding lengths with the aid of a Vernier calliper.

As an alternative, a graphics program can be used.

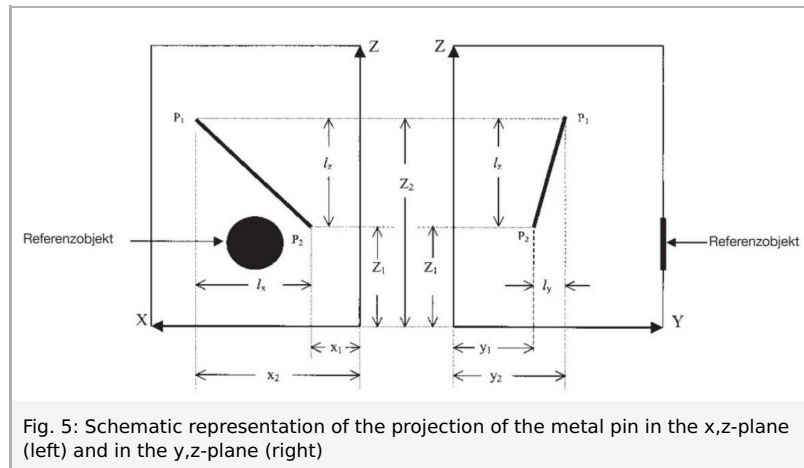


Fig. 5: Schematic representation of the projection of the metal pin in the x,z-plane (left) and in the y,z-plane (right)

Results (example):

The evaluation of the experiment example provided the following results:

$$l_x = 52.0 \text{ mm}, l_y = 35.0 \text{ mm}, l_z = 71.0 \text{ mm} \text{ and } V = 46.0/30.0 \text{ mm} = 1.533$$

$$\text{Thus: } l_x = 33.9 \text{ mm}, l_y = 22.8 \text{ mm}, l_z = 46.3 \text{ mm}$$

These value lead to: $l = 61.74 \text{ mm}$ and $l_w = 60.06 \text{ mm}$ (the actual length of the metal pin (as manufactured) is 60.0 mm).

Task 3: Determine the spatial position of the metal pin.

Based on the projection lengths l_x, l_y, l_z of l on the respective axes, the corresponding angles are calculated as follows:

$$\cos \alpha = \frac{l_x}{l}; \cos \beta = \frac{l_y}{l}; \cos \gamma = \frac{l_z}{l}$$

Using the values of the experiment example of task 2 ($l_x = 33.9 \text{ mm}, l_y = 22.8 \text{ mm}, l_z = 46.3 \text{ mm}$, and $V = \frac{46.0 \text{ mm}}{30.0 \text{ mm}} = 1.533$), the angles α , β , and γ are:

$$\alpha = \cos^{-1} \frac{l_x}{l} = 0.54 \rightarrow \alpha = 53.6^\circ$$

$$\beta = \cos^{-1} \frac{l_y}{l} = 0.40 \rightarrow \beta = 67.7^\circ$$

$$\gamma = \cos^{-1} \frac{l_z}{l} = 0.74 \rightarrow \alpha = 39.6^\circ$$